PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 3885–3896

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Effects of the Darcy number, the Prandtl number, and the Reynolds number on local thermal non-equilibrium

Sung Jin Kim *, Seok Pil Jang

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Taejon 305-701, South Korea

Received 22 August 2001; received in revised form 11 March 2002

Abstract

In the present study a general criterion for local thermal equilibrium is presented in terms of parameters of engineering importance which include the Darcy number, the Prandtl number, and the Reynolds number. For this, an order of magnitude analysis is performed for the case when the effect of convection heat transfer is dominant in a porous structure. The criterion proposed in this study is more general than the previous criterion suggested by Carbonell and Whitaker, because the latter is applicable only when conduction is the dominant heat transfer mode in a porous medium while the former can be applied even when convection heat transfer prevails. In order to check the validity of the proposed criterion for local thermal equilibrium, the forced convection phenomena in a porous medium with a microchanneled structure subject to an impinging jet are studied using a similarity transformation. The effects of the Darcy number, the Prandtl number, and the Reynolds number on local thermal non-equilibrium are systematically studied by comparing the temperature of the solid phase with that of the fluid phase as each of these parameters is varied. The proposed criterion is also validated with the existing experimental and numerical results for convection heat transfer in various porous materials that include some of the parameters used in the criterion such as a microchannel heat sink with a parallel flow, a packed bed, a cellular ceramic, and a sintered metal. It is shown that the criterion presented in this work well-predicts the validity of the assumption of local thermal equilibrium in a porous medium.

1. Introduction

Analysis of fluid flow and heat transfer in a porous medium has been a subject of continuous interest during the past decades because of the wide range of engineering applications. In addition to conventional applications such as solar receivers, building thermal insulation materials, packed bed heat exchangers, and energy storage units, investigators have found new applications in the emerging field of microscale heat transfer. As Bejan [1] pointed out, this new opportunity stems from the need for smaller flow passages and fins used in compact heat exchangers and electronics cooling. As the dimensions get smaller, classical flow structures approach a limiting case that is much better suited for porous medium modeling. Recently Koh and Colony [2] and Kim and his co-workers [3,4] modeled a microchannel heat sink as a porous medium. They showed that the analytical results for velocity and temperature distributions based on the porous medium modeling are in accord with numerical results obtained from Navier–Stokes and conventional energy equations.

In both conventional and emerging fields, various analytical and numerical studies on transport phenomena in a porous medium are generally based on the assumption of local thermal equilibrium; that is to say, both the fluid phase and the solid phase are at the same temperature. Under the assumption of local thermal equilibrium, many investigators have used the so-called one-equation model to obtain temperature distributions in a porous medium because an analysis using the one-equation model is simple and straightforward. However, the one-equation model is valid only when the temperature difference between the solid and the fluid phases is very small. When the condition of local thermal

* Corresponding author. Tel.: +82-42-869-3043; fax: +82-42-869-8207.

E-mail address: sungjinkim@kaist.ac.kr (S.J. Kim).

0017-9310/02/$ - see front matter
PII: S0017-9310(02)00109-6

Nomenclature

a_{sf}	interfacial surface area per unit volume
Bi	Biot number
d_p	pore diameter
Da	Darcy number
h_{sf}	interstitial heat transfer coefficient
H	height of microchannel heat sink
k	conductivity
k_{eff}	effective conductivity
K	permeability
l	characteristic length scale of pore size
L	characteristic length scale of system
$\dot{m}$	mass flow rate
Nu	Nusselt number
p	pressure
Pr	Prandtl number
Pr_{eff}	effective Prandtl number
q	heat transfer rate from solid phase to fluid phase in a representative elementary volume
Q	total heat transfer rate by the fluid flow through a porous medium
R_k	effective thermal conductivity ratio
Re	Reynolds number
t	time scale
T	temperature
u	velocity along the x-direction
u_p	pore velocity
v	velocity along the y-direction
V	total system volume
V_0	impinging velocity
W	width of heat sink
W_c	channel width for fluid region

Greek symbols

α	thermal diffusivity
Δ	difference
ε	porosity
η	similarity variable
ν	dynamic viscosity
θ	dimensionless temperature
ρ	density
Ψ	stream function

Subscripts/superscripts

f	fluid
eff	effective
l	characteristic length scale of pore size
L	characteristic length scale of system
s	solid
w	wall
∞	top surface

Special symbol

$\langle\,\rangle$	volume-averaged value

equilibrium is far from reality, the one-equation model needs to be replaced with the two-equation model, which treats the solid phase and the fluid phase separately. But an analysis of heat transfer in a porous medium based on the two-equation model is more involved because it requires information on the interstitial heat transfer coefficient between the fluid phase and the solid phase as well as the interfacial surface area. Due to this difficulty, most investigators have used the one-equation model to obtain velocity and temperature distributions in a porous medium without examining the validity of the assumption of local thermal equilibrium.

Whitaker and his co-workers [5–8] performed a pioneering work on local thermal equilibrium. They presented a criterion for the validity of the assumption of local thermal equilibrium using an order of magnitude analysis. Their criterion is proposed in the case where the effect of conduction is dominant in a representative elementary volume (REV) enclosing both the fluid and the solid phases. However, heat transfer mostly occurs by convection through the pores in a porous medium. In addition, it is difficult to directly apply their criterion to engineering problems because it has several coefficients which are awkward to define, as pointed out by Vafai and Amiri [9]. Amiri and Vafai [10] obtained the local temperature distributions of the fluid phase and the solid phase from numerical results for forced convection through a channel filled with packed beds. They presented an error contour map in terms of the particle Reynolds number, the Darcy number, and the thermal diffusivity ratio based on the qualitative ratings. Nield [11] and Nield and Kuznetsov [12] presented the conditions of local thermal non-equilibrium in a saturated porous channel using an analytical solution for velocity and temperature fields. Nield [11] concluded that the effect of local thermal non-equilibrium is to reduce the Nusselt number at the interface between the fluid and solid phases. Lee and Vafai [13] proposed a criterion for the validity of the one-equation model in the case of flow through a porous channel subjected to a constant heat flux on the top and bottom walls by using analytical solutions based on the Darcian flow model. They focused on the qualitative presentation of the heat transfer in porous media by taking the effective conductivities and the interstitial heat transfer coefficient as known parameters without referring to a specific geometry or the Reynolds number. Kim et al. [4] showed that the assumption of local thermal equilibrium in a microchannel heat sink, which is modeled as a porous medium, is valid as the Darcy number approaches zero and the effective thermal conductivity ratio infinity. Even though studies on local thermal equilibrium have been conducted for many years, a general criterion for the validity of the local thermal equilibrium assumption in terms of engineering parameters such as the Darcy number, the Prandtl number, and the Reynolds number has not been available, to the authors' knowledge.

The purpose of the present study is to present a general criterion for local thermal equilibrium in terms of parameters of engineering importance which include the Darcy number, the Prandtl number, and the Reynolds number. For this, an order of magnitude analysis is performed for the case when the effect of convection heat transfer is dominant in a porous structure. The criterion proposed in this study is more general than the previous criterion suggested by Carbonell and Whitaker, because the latter is applicable only when conduction is the dominant heat transfer mode in a porous medium while the former can be applied even when convection heat transfer prevails. In order to check the validity of the proposed criterion for local thermal equilibrium, the forced convection phenomena in a porous medium with a microchanneled structure subject to an impinging jet are studied using a similarity transformation. The effects of the Darcy number, the Prandtl number, and the Reynolds number on local thermal non-equilibrium are systematically studied by comparing the temperature of the solid phase with that of the fluid phase as each of these parameters is varied. The reason that the proposed criterion is benchmarked against the problem of forced convection in a porous medium with a microchannel structure subject to an impinging jet is because the chosen problem contains all the engineering parameters involved in the criterion. The proposed criterion is also validated with the existing experimental and numerical results for convection heat transfer in various porous materials that include some of the parameters used in the criterion such as a microchannel heat sink with a parallel flow, a packed bed, a cellular ceramic, and a sintered metal.

2. Criterion for local thermal equilibrium

The assumption of local thermal equilibrium is valid when the temperature difference between the solid phase and the fluid phase in an REV is much smaller than that occurring over the system dimension [14]

$$\Delta T_L \gg \Delta T_l \tag{1}$$

where ΔT_L and ΔT_l are the temperature difference occurring over the dimension of the system and the temperature difference between the solid phase and fluid phase in an REV, respectively. In order to estimate both ΔT_L and ΔT_l in a porous medium, we use an order of magnitude analysis. Each temperature difference is depicted in Fig. 1. It is assumed that the heat transfer rate from the solid phase to the fluid phase in the overall system is equal to the heat transfer rate which is carried by the fluid flowing through the porous medium. The local heat transfer rate in the REV can be expressed as

$$q \sim h_{\mathrm{sf}} a_{\mathrm{sf}} \Delta T_l \tag{2}$$

where $\Delta T_l = T_{\mathrm{s}} - T_{\mathrm{f}}$. Hence the heat transfer rate from solid phase to fluid phase in the overall system is

$$Q \sim h_{\mathrm{sf}} a_{\mathrm{sf}} V \Delta T_l \tag{3}$$

The amount of heat, which is transferred to or from the fluid flowing through the porous medium, can be expressed as

$$Q \sim \dot{m} C_{\mathrm{p}} \Delta T_L \tag{4}$$

where $\Delta T_L = T_{\mathrm{f,outlet}} - T_{\mathrm{f,inlet}}$. In the above equations a_{sf}, C_{p}, h_{sf}, $\dot{m}$, V, q, and Q are interfacial surface area per unit volume, specific heat, interstitial heat transfer coefficient between the fluid phase and the solid phase, mass flow rate, system volume, heat transfer rate from solid phase to fluid phase in the REV, and heat transfer rate by the fluid flow through the porous medium, respectively. Each temperature difference is obtained from Eqs. (3) and (4) as

$$\Delta T_l \sim \frac{Q}{h_{\mathrm{sf}} a_{\mathrm{sf}} V} \tag{5}$$

$$\Delta T_L \sim \frac{Q}{\dot{m} C_{\mathrm{p}}} \tag{6}$$

By substituting Eqs. (5) and (6) into Eq. (1), the criterion for local thermal equilibrium is expressed by the following equation:

$$\frac{\dot{m} C_{\mathrm{p}}}{h_{\mathrm{sf}} a_{\mathrm{sf}} V} \ll 1 \tag{7}$$

Since $\dot{m} = \rho \varepsilon V / t$, Eq. (7) can be rearranged as

$$\frac{(\rho C_{\mathrm{p}})_{\mathrm{f}} \varepsilon}{h_{\mathrm{sf}} a_{\mathrm{sf}} t} \ll 1 \tag{8}$$

This criterion implies that the effect of local thermal equilibrium becomes dominant in a porous medium as either the interstitial heat transfer coefficient or the interfacial surface area for heat transfer increases. This trend is similar to what Nield concluded in his paper, i.e., the effect of local thermal non-equilibrium decreases the Nusselt number at the interface between the fluid phase and solid phase [11,12]. On the other hand, the criterion presented by Carbonell and Whitaker [5] is expressed as

$$\frac{\varepsilon (\rho C_{\mathrm{p}})_{\mathrm{f}} l^2}{t} \left(\frac{1}{k_{\mathrm{f}}} + \frac{1}{k_{\mathrm{s}}} \right) \ll 1 \tag{9}$$

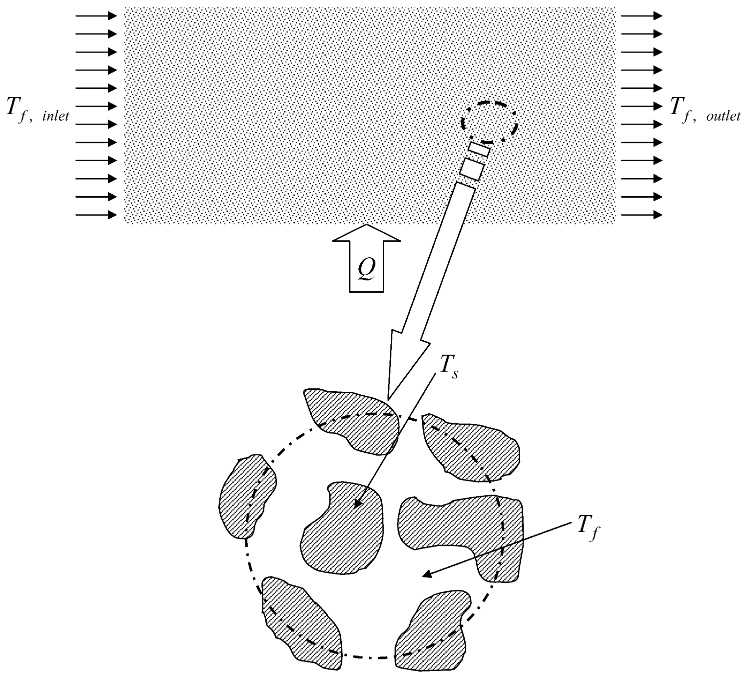

Fig. 1. Schematic diagram of a system and a representative elementary volume.

where ε, ρ, C_p, l, t, k_f, and k_s denote porosity, fluid density, fluid specific heat, characteristic length scale of pore size, time scale, fluid conductivity, and solid conductivity, respectively. As pointed out earlier, Eq. (9) is derived for the case where conduction is the dominant heat transfer mode. As a result, Eq. (9) did not include the effect on local thermal equilibrium of convection heat transfer between the fluid phase and the solid phase. Therefore, Eq. (9) is no longer applicable for determining the validity of the assumption of local thermal equilibrium in a porous medium if convection heat transfer is dominant. The criterion given in Eq. (8) can be more easily applicable to an analysis of fluid flow and heat transfer in a porous medium if it is expressed in parameters of engineering importance. The interstitial heat transfer coefficient, the interfacial surface area per unit volume and the time scale used in Eq. (8) can be expressed by

$$h_{sf} \sim \frac{Nuk_f}{d_p}, \quad a_{sf} \sim \frac{\varepsilon}{d_p}, \quad t \sim \frac{L}{u_p}, \tag{10}$$

where Nu, k_f, d_p, ε, u_p and L are Nusselt number, fluid conductivity, characteristic length for pore size, porosity, pore velocity, and characteristic length for the system. By substituting Eq. (10) into Eq. (8), the criterion for local thermal equilibrium is presented by the following equation:

$$Pr_{eff,f} Re_{d_p} Da^{1/2} \frac{\varepsilon}{Nu} \ll 1 \tag{11}$$

where the effective fluid Prandtl number, the Reynolds number, the Darcy number, the Nusselt number, and the effective thermal diffusivity are defined respectively as

$$Pr_{eff,f} = \frac{\nu}{\alpha_{eff,f}}, \quad Re_{d_p} = \frac{u_p d_p}{\nu}, \quad Da \sim \frac{d_p^2}{L^2},$$

$$Nu = \frac{h_{sf} d_p}{k_f}, \quad \alpha_{eff,f} = \frac{(\rho C_p)_f}{k_{eff,f}} = \frac{(\rho C_p)_f}{\varepsilon k_f}$$

In addition, the effective fluid Prandtl number is expressed as

$$Pr_{eff,f} = \frac{\nu}{\alpha_{eff,f}} = \frac{(\rho C_p)_f \nu}{k_{eff,s}} \frac{1}{R_k}, \quad R_k = \frac{k_{eff,f}}{k_{eff,s}} \tag{12}$$

where R_k is the effective thermal conductivity ratio.

From the criterion, Eq. (11), which is expressed in terms of important engineering parameters, the effect of local thermal equilibrium in a porous medium is shown to become stronger as either any one of the Reynolds number, the Prandtl number or the Darcy number decreases or the Nusselt number increases. In addition, as the effective thermal conductivity ratio increases, the condition of local thermal equilibrium also exists because the Prandtl number is inversely proportional to the effective thermal conductivity ratio.

3. Similarity solution for the forced convection in a porous medium with a microchannel structure subjected to an impinging jet

3.1. Problem description and modeling

In order to check the validity of the proposed criterion, Eq. (11), for local thermal equilibrium, the forced convection phenomena in a porous medium with a microchannel structure subject to an impinging jet are studied using a similarity transformation. The reason that the proposed criterion is benchmarked against the problem of forced convection in a porous medium with a microchannel structure subject to an impinging jet is because this problem contains all the engineering parameters involved in the criterion.

The problem under consideration in this paper is an impinging flow through a microchannel heat sink as shown in Fig. 2(a). The fluid impinges on the microchannel heat sink along the y-axis and then flows parallel to the x-axis. The bottom surface is kept constant at a high temperature and the top surface at a low temperature. This thermal boundary condition is used to examine the effect of local thermal equilibrium in this problem because if there is some agency that forces the volume-averaged fluid temperature to be different from the corresponding volume-averaged solid temperature on the boundaries of a porous medium, then local thermal non-equilibrium may automatically be present, as pointed out by Nield [11]. A coolant takes heat away from a heat-dissipating component attached to a microchannel heat sink. In analyzing the problem, all physical properties are assumed to be constant.

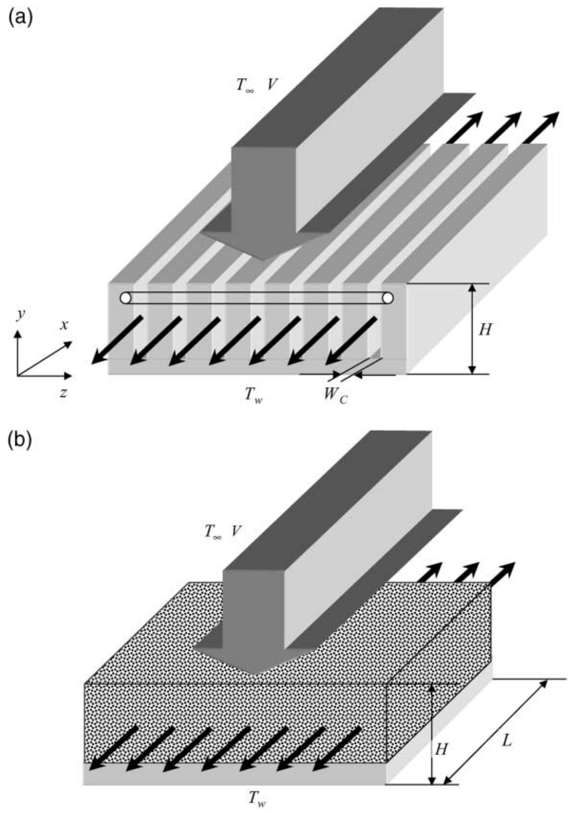

Fig. 2. Porous medium approach: (a) microchannel heat sink subject to an impinging jet and (b) equivalent porous medium.

As shown in Fig. 2(b), the microchannel heat sink subject to an impinging jet is modeled as a porous medium based on the idea proposed by Koh and Colony [2]. The governing equations are established by applying the volume-averaging technique. In order to analyze the fluid flow in a porous medium with the boundary effect included, the Brinkman-extended Darcy equation is used. Regarding the temperature profiles, both the one-equation model and the two-equation model are employed. The one-equation model is based on the assumption that both the fluid phase and the solid phase are at the same temperature within an REV while the two-equation model treats the fluid phase and the solid phase separately within an REV. The general criterion of local thermal equilibrium is verified by results obtained from the two-equation model.

In this paper, as shown in Fig. 2(a), the REV for volume-averaging is a slender cylinder aligned parallel to the bottom surface of the microchannel heat sink but perpendicular to the flow direction. The resultant volume-averaged equations are valid because the REV is long enough to yield meaningful averages, and the direction of volume-averaging does not strongly depend on the paths of fluid flow and heat transfer as pointed out by Kim et al. [4].

3.2. Velocity distribution

To obtain the velocity profile in a microchannel heat sink subject to an impinging jet, the Brinkman-extended Darcy equation is solved. The Brinkman-extended Darcy equations are given as follows:

X momentum equation:

$$\langle u\rangle_{\mathrm{f}}\frac{\partial\langle u\rangle_{\mathrm{f}}}{\partial x}+\langle v\rangle_{\mathrm{f}}\frac{\partial\langle u\rangle_{\mathrm{f}}}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu_{\mathrm{f}}\left[\frac{\partial^2\langle u\rangle_{\mathrm{f}}}{\partial x^2}+\frac{\partial^2\langle u\rangle_{\mathrm{f}}}{\partial y^2}\right]-\frac{\varepsilon}{K}\langle u\rangle_{\mathrm{f}}\nu_{\mathrm{f}} \tag{13}$$

Y momentum equation:

$$\langle u\rangle_{\mathrm{f}}\frac{\partial\langle v\rangle_{\mathrm{f}}}{\partial x}+\langle v\rangle_{\mathrm{f}}\frac{\partial\langle v\rangle_{\mathrm{f}}}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu_{\mathrm{f}}\left[\frac{\partial^2\langle v\rangle_{\mathrm{f}}}{\partial x^2}+\frac{\partial^2\langle v\rangle_{\mathrm{f}}}{\partial y^2}\right]-\frac{\varepsilon}{K}\langle v\rangle_{\mathrm{f}}\nu_{\mathrm{f}}, \tag{14}$$

$$\langle u\rangle_{\mathrm f}=\langle v\rangle_{\mathrm f}=0 \ \text{at}\ y=0,\quad \langle v\rangle_{\mathrm f}=V_0\ \text{at}\ y=H$$

where $\langle\rangle_{\mathrm f}$ denotes a volume-averaged value over the fluid region and p, $\nu_{\mathrm f}$, u, v, ε, K, V_0, and H are pressure, fluid dynamic viscosity, velocity along the x direction and the y direction, porosity, permeability, impinging velocity at the top surface of the microchannel heat sink and the height of the microchannel heat sink.

In order to transform Eqs. (13) and (14) into a simpler form, we assume that the type of impinging flow in a porous medium is similar to that of an impinging flow over a flat plate immersed in a regular fluid. It can be assumed that the stream function of an impinging flow in a porous medium is a fractional function. A similarity variable for an impinging flow in a porous medium is deduced from the similarity variable for the impinging flow in a regular fluid. As a result, partial differential momentum equations can be transformed into an ordinary differential equation with the similarity variable. The similarity variable, η and the stream function Ψ, are given by

$$\eta=y\sqrt{\frac{B}{\nu_{\mathrm f}}} \tag{15}$$

$$\Psi=\sqrt{B\nu_{\mathrm f}}F(\eta)x \tag{16}$$

$$B=\frac{V_0}{H} \tag{17}$$

With Eqs. (15) and (16), the momentum equation can be simplified as

$$F'''+FF''-F'^2-\frac{\varepsilon}{K}\frac{\nu_{\mathrm f}}{B}F'+\frac{\nu_{\mathrm f}}{K}\frac{\varepsilon}{B}+1=0, \tag{18}$$

$$F=F'=0\ \text{at}\ \eta=0,\quad F'=1\ \text{at}\ \eta\to\infty$$

The porosity and the permeability presented by Kim and Kim [3] for the rectangular microchannel can be represented as

$$\varepsilon=\frac{W_{\mathrm c}}{W},\quad K=\frac{\varepsilon W_{\mathrm c}^2}{12} \tag{19}$$

where $W_{\mathrm c}$ and W are channel width for fluid region and width of heat sink. Similarity solutions to Eq. (18) are obtained for velocity distributions numerically using the shooting method.

3.3. The one-equation model

In the one-equation model, the governing equation can be obtained based on the assumption that the temperature of the fluid phase and that of the solid phase in an REV are the same, i.e., $\langle T\rangle_{\mathrm f}=\langle T\rangle_{\mathrm s}=\langle T\rangle$. The volume-averaged energy equation for the one-equation model is given by

$$\varepsilon\langle\rho C_{\mathrm p}\rangle_{\mathrm f}\left(\langle u\rangle_{\mathrm f}\frac{\partial\langle T\rangle}{\partial x}+\langle v\rangle_{\mathrm f}\frac{\partial\langle T\rangle}{\partial y}\right)=k_{\mathrm{eff}}\left(\frac{\partial^2\langle T\rangle}{\partial x^2}+\frac{\partial^2\langle T\rangle}{\partial y^2}\right), \tag{20}$$

$$\langle T\rangle=T_{\mathrm w}\ \text{at}\ y=0,\quad \langle T\rangle=T_\infty\ \text{at}\ y=H$$

where ρ, $C_{\mathrm p}$, k_{eff}, $T_{\mathrm w}$, and are T_∞ density, specific heat, effective thermal conductivity, and temperatures of the bottom surface and the top surface, respectively. The effective thermal conductivity is obtained assuming parallel arrangement of the porous structure.

For impinging flow over a flat plate, Goldstein suggested that a similarity solution for the energy equation exists and is function of the y-axis if the wall temperature and the temperature of the impinging flow at the top are constant [15]. In order to obtain temperature distributions in a microchannel heat sink modeled as a porous medium, we use the modified similarity variable and dimensionless variable. A dimensionless temperature θ is defined such that it is zero at the bottom surface and unity at the top surface,

$$\theta=\frac{\langle T\rangle-T_{\mathrm w}}{T_\infty-T_{\mathrm w}},\quad \eta=y\sqrt{\frac{B}{\nu_{\mathrm f}}} \tag{21}$$

Substituting u and v obtained from the solution of Eq. (18) into the energy Eq. (20) and using the similarity variable and the dimensionless temperature gives a second-order ordinary differential equation:

$$\theta''+\varepsilon F Pr_{\mathrm{eff}}\theta'=0 \tag{22}$$

$$Pr_{\mathrm{eff}}=\frac{\mu_f C_{\mathrm{p,f}}}{k_{\mathrm{eff}}}$$

$$\theta=0\ \text{at}\ \eta=0,\quad \theta=1\ \text{at}\ \eta\to\infty$$

where Pr_{eff} is the effective Prandtl number.

The analytical solution can be obtained from Eq. (22):

$$\theta=\frac{\int_0^\eta \exp\left(-Pr_{\mathrm{eff}}\int_0^\eta F(\eta)\varepsilon\,\mathrm d\eta\right)\mathrm d\eta}{\int_0^\infty \exp\left(-Pr_{\mathrm{eff}}\int_0^\eta F(\eta)\varepsilon\,\mathrm d\eta\right)\mathrm d\eta} \tag{23}$$

3.4. The two-equation model

The volume-averaged energy equations and the boundary conditions for the solid phase and the fluid phase without the assumption for local thermal equilibrium are expressed, respectively, as

$$k_{\mathrm{eff,s}}\left(\frac{\partial^2\langle T\rangle_{\mathrm s}}{\partial y^2}\right)+h_{\mathrm{sf}}a_{\mathrm{sf}}\left(\langle T\rangle_{\mathrm f}-\langle T\rangle_{\mathrm s}\right)=0,\ \text{in the solid phase} \tag{24}$$

$$\varepsilon<\rho C_{\mathrm p}>\left(\langle v\rangle_{\mathrm f}\frac{\partial\langle T\rangle_{\mathrm f}}{\partial y}\right)=k_{\mathrm{eff,f}}\left(\frac{\partial^2\langle T\rangle_{\mathrm f}}{\partial y^2}\right)+h_{\mathrm{sf}}a_{\mathrm{sf}}\left(\langle T\rangle_{\mathrm s}-\langle T\rangle_{\mathrm f}\right),\ \text{in the fluid phase} \tag{25}$$

where

$$k_{\mathrm{eff,s}} = (1-\varepsilon)k_{\mathrm{s}}, \quad k_{\mathrm{eff,f}} = \varepsilon k_{\mathrm{f}}, \quad a_{\mathrm{sf}} = \frac{2}{W_{\mathrm{c}}}$$

$$\langle T\rangle_{\mathrm{s}} = \langle T\rangle_{\mathrm{f}} = T_{\mathrm{w}} \ \text{at}\ y = 0,$$
$$\langle T\rangle_{\mathrm{s}} = \langle T\rangle_{\mathrm{f}} = T_{\infty} \ \text{at}\ y = H$$

and a_{sf}, $k_{\mathrm{eff,s}}$, k_{s}, $k_{\mathrm{eff,f}}$, k_{f}, and h_{sf} are interfacial surface area per unit volume, effective solid thermal conductivity, solid conductivity, effective fluid conductivity, fluid conductivity, and interstitial heat transfer coefficient, respectively.

Eqs. (24) and (25) can be non-dimensionalized by using the similarity variable in Eq. (15) and the following dimensionless variables:

$$Bi = \frac{h_{\mathrm{sf}} a_{\mathrm{sf}} H^2}{k_{\mathrm{eff,s}}}, \quad Da = \frac{W_{\mathrm{c}}^2}{12H^2}, \quad Pr_{\mathrm{eff,f}} = \frac{\mu_{\mathrm{f}} C_{\mathrm{p,f}}}{k_{\mathrm{eff,f}}},$$
$$R_k = \frac{k_{\mathrm{eff,f}}}{k_{\mathrm{eff,s}}}, \quad Re_{W_{\mathrm{c}}} = \frac{V_0 W_{\mathrm{c}}}{\nu_{\mathrm{f}}} \qquad (26)$$

where Bi, Da, $Pr_{\mathrm{eff,f}}$, R_k, and $Re_{W_{\mathrm{c}}}$ are equivalent Biot number, Darcy number, effective fluid Prandtl number, effective thermal conductivity ratio, and Reynolds number. The dimensionless governing equations and boundary conditions for the two-equation model are expressed as the following equations:

$$\theta_{\mathrm{s}}'' + Bi\frac{1}{Re_{W_{\mathrm{c}}}}(12Da)^{1/2}(\theta_{\mathrm{f}} - \theta_{\mathrm{s}}) = 0, \ \text{in the solid phase} \qquad (27)$$

$$\theta_{\mathrm{f}}'' + \varepsilon Pr_{\mathrm{f}} F\theta_{\mathrm{f}}' + Bi\frac{1}{R_k}\frac{1}{Re_{W_{\mathrm{c}}}}(12Da)^{1/2}(\theta_{\mathrm{s}} - \theta_{\mathrm{f}}) = 0, \ \text{in the fluid phase} \qquad (28)$$

$$\theta_{\mathrm{f}} = \theta_{\mathrm{s}} = 0 \ \text{at}\ \eta = 0, \quad \theta_{\mathrm{f}} = \theta_{\mathrm{s}} = 1 \ \text{at}\ \eta \to \infty$$

With these boundary conditions, the temperature distributions for both fluid and solid phases are obtained by the finite volume method.

3.5. Discussion on the velocity and the temperature distributions

Recently investigators have proven that results obtained from a porous medium approach agree well with analytical solutions as well as numerical results. In particular, Koh and Colony [2] and Kim et al. [3,4] studied forced convection through a microchannel heat sink modeled as a porous medium. In order to validate the porous medium model for the microchannel heat sink, they compared the analytical solutions from the conventional momentum equation and the numerical solutions from the conventional energy equation with results obtained from the porous medium approach. They showed that results obtained from the porous medium approach are in close agreement with the volume-averaged velocity and temperature distributions obtained from analytical and numerical solutions.

In the present study, in order to validate the solutions obtained using a porous medium approach, the results from similarity solutions are compared with volume-averaged velocity and temperature distributions for the conjugate heat transfer problem comprising both the solid and fluid regions. The conventional momentum and energy equations are solved numerically with the finite volume method.

In Fig. 3, the velocity and temperature distributions obtained using a porous medium approach are shown to predict quite well the corresponding volume-averaged velocity and temperature distributions obtained numerically. These similarity solutions obtained from the porous medium approach are helpful in obtaining velocity and temperature profiles and identifying the effects of engineering parameters on local thermal equilibrium rather easily.

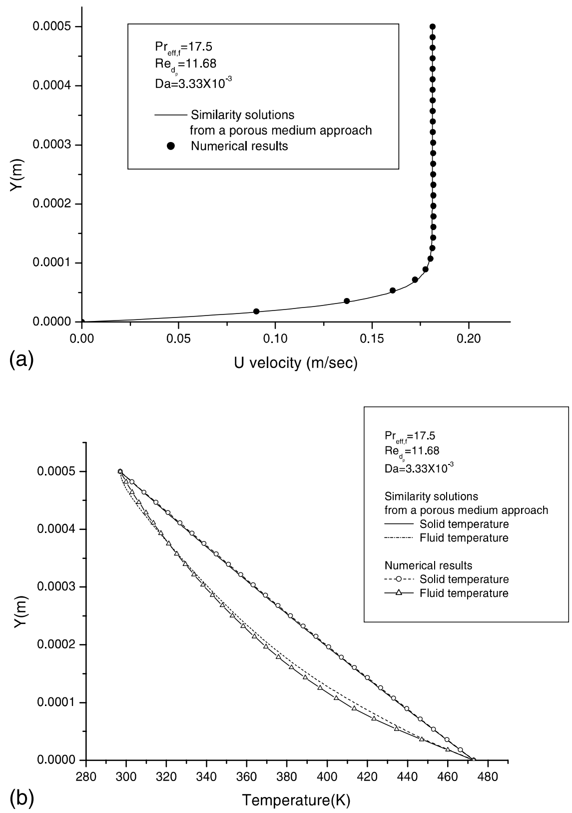

Fig. 3. Velocity and temperature distribution.

4. Results and discussion on the criterion

4.1. The microchannel heat sink with an impinging jet

The effects of engineering parameters on local thermal non-equilibrium are investigated by comparing volume-averaged temperature distributions of the solid phase with those of the fluid phase, both of which are obtained from the two-equation model for a microchannel heat sink subject to an impinging jet. In the microchannel heat sink subject to an impinging jet, the Nusselt number can be represented by the following correlation:

$$Nu = 1.2434 Re_{d_p}^{0.1368} Pr_{\mathrm{eff,f}}^{0.161} + 2.0 \tag{29}$$

which is obtained from numerical results in Section 3. In Fig. 4, it is shown that this correlation corresponds to numerical results within 1.0%.

In order to validate the criterion for local thermal equilibrium and to understand the effects of engineering parameters on local thermal non-equilibrium, limiting cases for the microchannel heat sink subject to an impinging jet are considered as follows:

4.1.1. Limiting case 1 ($Pr_{eff,f} \to 0$)

As $Pr_{\mathrm{eff,f}}$ approaches zero, the Nusselt number expressed as Eq. (29) becomes constant at 2.0. Then, the criterion proposed previously as Eq. (11) is met. Therefore, the assumption of local thermal equilibrium is valid in this case. This is also the case as the effective thermal conductivity ratio is increased, because the effective thermal conductivity ratio is inversely proportional to $Pr_{\mathrm{eff,f}}$ as shown in Eq. (12). On the other hand, as $Pr_{\mathrm{eff,f}}$ is increased, the assumption of local thermal equilibrium becomes less and less valid.

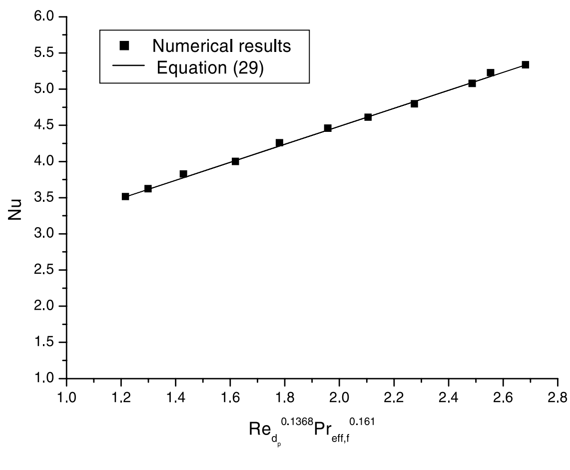

Fig. 4. Nusselt number in the microchannel heat sink subject to an impinging jet.

4.1.2. Limiting case 2 ($Re_{d_p} \to 0$)

As Re_{d_p} approaches zero, the Nusselt number expressed as Eq. (29) becomes constant at 2.0. Then, the criterion proposed previously as Eq. (11) is met. Therefore, the assumption of local thermal equilibrium is also valid in this case. On the other hand, as Re_{d_p} is increased, the assumption of local thermal equilibrium becomes less and less valid.

4.1.3. Limiting case 3 ($Da \to 0$)

As Da approaches zero, the criterion expressed as Eq. (11) approaches 0. So, the assumption of local thermal equilibrium is also valid in this case. On the other hand, as is Da increased, the assumption of local thermal equilibrium becomes less and less valid.

From results of these limiting cases, the assumption of local thermal equilibrium in a microchannel heat sink subject to an impinging jet is shown to be reasonable when any one of the Prandtl number, the Reynolds number, or the Darcy number approaches zero. In order to validate these qualitative results discussed in the limiting cases, the volume-averaged temperature profiles of the solid phase are compared quantitatively with those of the fluid phase, both of which are obtained from the two-equation model. In Figs. 5–7, it is shown that the volume-averaged temperature distributions of the solid phase are nearly identical to those of the fluid phase as either $Pr_{\mathrm{eff,f}} \to 0$, $Re_{d_p} \to 0$, or $Da \to 0$. On the other hand, as $Pr_{\mathrm{eff,f}}$, Re_{d_p}, or Da is increased, the volume-averaged temperature profiles of the solid phase becomes more and more different from those of the fluid phase. In other words, as either $Pr_{\mathrm{eff,f}}$, Re_{d_p}, or Da approaches zero, the effects of local thermal equilibrium are dominant because conduction is dominant in the microchannel heat sink with an impinging jet. On the other hand, as $Pr_{\mathrm{eff,f}}$, Re_{d_p}, and Da are increased, the assumption of local thermal equilibrium become less and less valid. To quantify the outcome based on qualitative ratings for the assumption of local thermal equilibrium, the percentage error is defined as

$$\mathrm{Error}(\%) = \left[\frac{\mathrm{Max}(\langle T \rangle_{\mathrm{s}} - \langle T \rangle_{\mathrm{f}})}{T_{\mathrm{w}} - T_{\infty}} \right] \times 100 \tag{30}$$

The percentage error are presented in Fig. 8 in terms of the value of the left-hand side of the proposed criterion. As shown in Fig. 8, the percentage error increases with the increasing value of the left-hand side of the proposed criterion. When the value of the left-hand side of the proposed criterion is of the order of 10^{-2}, the percentage error is less than 10%. When it is of the order of 10^{-1}, the percentage error is less than 35%. When it is of the order of one, the percentage error is more than 35%.

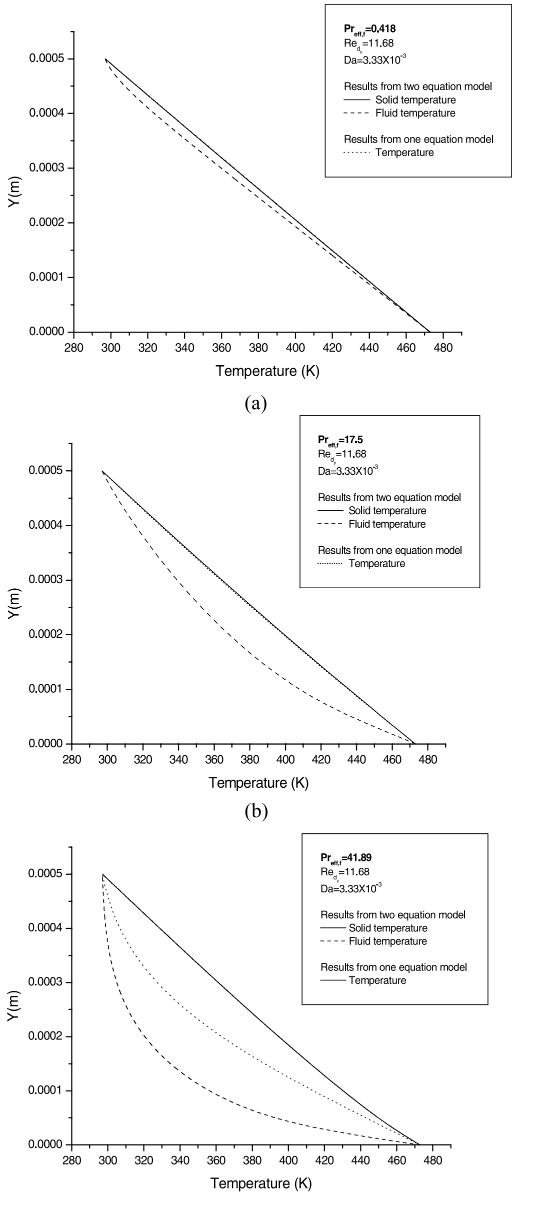

Fig. 5. Effect of the Prandtl number on temperature distributions: (a) $Pr_{\mathrm{eff,f}} = 0.418$, (b) $Pr_{\mathrm{eff,f}} = 17.5$, and (c) $Pr_{\mathrm{eff,f}} = 41.89$.

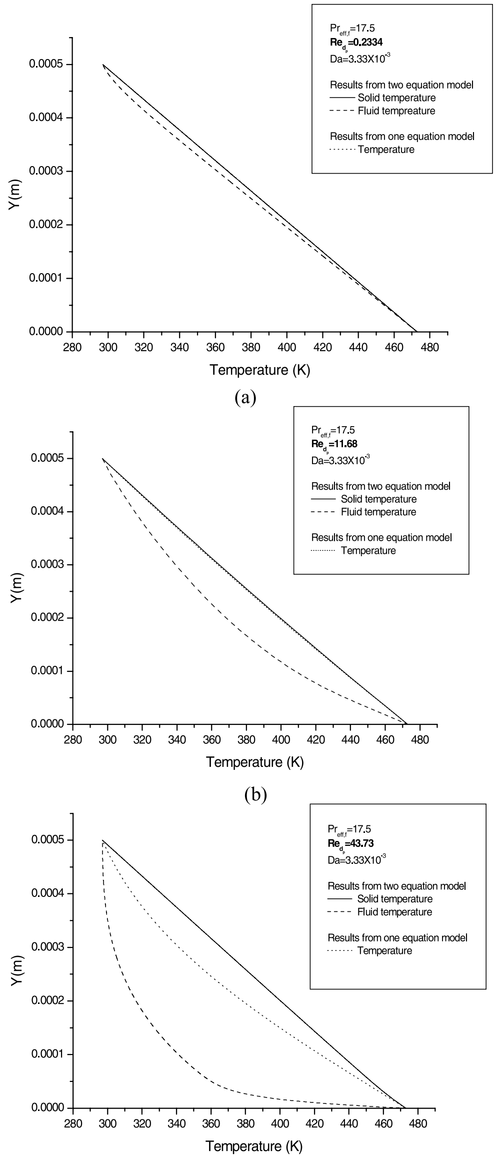

Fig. 6. Effect of the Reynolds number on temperature distributions: (a) $Re_{d_\mathrm{p}} = 0.2334$, (b) $Re_{d_\mathrm{p}} = 11.68$, and (c) $Re_{d_\mathrm{p}} = 43.73$.

4.2. The microchannel heat sink and the channel filled with a porous medium

In order to confirm the proposed criterion for local thermal equilibrium generally, the criterion is applied to other cases such as the microchannel heat sink presented by Kim et al. [4] and the channel filled with a porous medium presented by Amiri and Vafai [10].

Kim et al. [4] checked the validity of local thermal equilibrium in the microchannel heat sink modeled as a porous medium in terms of the Darcy number and the effective thermal conductivity ratio. Because both the Nusselt number and the Reynolds number are assumed to be constant for fully developed flow in the microchannel heat sink with a parallel flow, the condition of local thermal equilibrium depends on the Darcy number and the effective thermal conductivity ratio. They concluded that the assumption of local thermal equilibrium is valid as the Darcy number is decreased or the effective

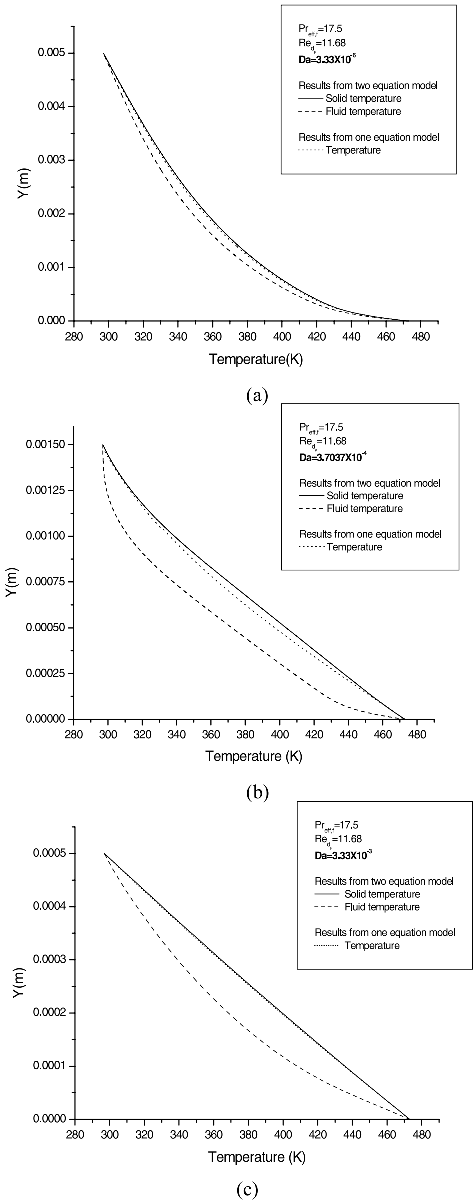

Fig. 7. Effect of the Darcy number on temperature distributions: (a) $Da = 3.33 \times 10^{-6}$, (b) $Da = 3.7037 \times 10^{-4}$, and (c) $Da = 3.33 \times 10^{-3}$.

thermal conductivity is increased. The same conclusion can be drawn from Eq. (11).

Amiri and Vafai [10] compared the local temperature distributions of the fluid phase with those of the solid phase from numerical results in the case of fluid flow through channel filled with a packed bed. In order to solve the energy equations for the two-equation model, they used the Nusselt number based on the interstitial heat transfer coefficient presented by Wakao et al. [16].

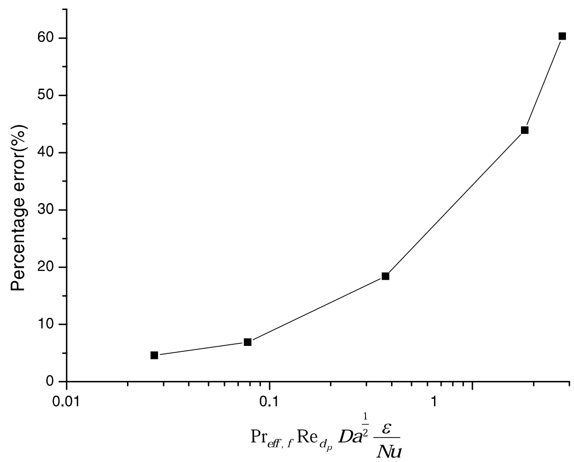

Fig. 8. Value of the criterion in a microchannel heat sink subject to an impinging jet.

$$Nu = \frac{h_{sf} d_p}{k_f} = 2 + 1.1 Pr^{1/3} Re_{d_p}^{0.6} \tag{31}$$

where d_p is the pore diameter. They reported that the assumption of local thermal equilibrium is valid as either the Reynolds number or the Darcy number approaches zero. The results presented by Amiri and Vafai [10] match with those predicted qualitatively by Eq. (11).

4.3. The other porous media

For a sintered metal and a cellular ceramic used in industrial applications, the assumption of local thermal equilibrium is also valid when any one of the Reynolds number, the Darcy number or the Prandtl number approaches zero. However, the sintered metal and the cellular ceramic exhibit different characteristics with respect to local thermal equilibrium compared with a packed bed, a microchannel heat sink with a parallel flow and a microchannel heat sink with an impinging jet, as will be shown below. For the sintered metal, Kar and Dybbs [17] as well as Maiorov et al. [18] suggested the Nusselt number based on the interstitial heat transfer coefficient as

$$Nu = \frac{h_{sf} d_p}{k_f} \sim C Re_{d_p}^n \quad (0 < Re_{d_p} < 10^2) \tag{32}$$

$n \approx 1.35$ in Kar and Dybbs [17],

$$Nu = \frac{h_{sf} d_p}{k_f} \sim C Re_{d_p}^n \quad (0 < Re_{d_p} < 10^3) \tag{33}$$

$0.65 < n < 1.84$ in Maiorov et al. [18].

For the cellular ceramic, the Nusselt number based on the interstitial heat transfer coefficient is presented by Fu et al. [19] as

$$Nu = \frac{h_{sf} d_p}{k_f} \sim C Re_{d_p}^n \quad (0 < Re_{d_p} < 10^3) \tag{34}$$

$0.9 < n < 1.18$ in Fu et al. [19].

In these cases, the proposed criterion is met if and only if $n > 1$ and $Re_{d_p} \to \infty$ when substituting one of Eqs. (32)–(34) into Eq. (11). Therefore, the assumption of local thermal equilibrium is valid in these cases even if the Reynolds number is increased. As mentioned before, this result is different in that local thermal equilibrium is observed as the Reynolds number is increased, while the assumption of local thermal equilibrium is shown in Sections 4.1 and 4.2 to be valid as the Reynolds number is decreased for the microchannel heat sink and the packed bed. This means physically that the assumption of local thermal equilibrium is valid even when convection as a heat transfer mode is dominant in the sintered metal and the cellular ceramic. This results from the fact that ΔT_l between the solid phase and the fluid phase in an REV decreases faster than the ΔT_L occurring in the fluid flow through the porous medium if the interstitial heat transfer coefficient is proportional to the Reynolds number with a power greater than 1. Consequently, the assumption of local thermal equilibrium is valid even if convection is dominant in a porous medium such as a sintered metal and a cellular ceramic for which the interstitial heat transfer coefficient is proportional to the Reynolds number with a power greater than 1.

4.4. Summary

It is shown that the effects of the Darcy number, the Prandtl number, and the Reynolds number on local thermal equilibrium in the microchannel heat sink subjected to an impinging jet are well-predicted by the criterion proposed in this study. The proposed criterion is also validated with the existing experimental and numerical results for forced convection heat transfer in various porous materials that include some of the parameters used in the criterion such as a microchannel heat sink with a parallel flow, a packed bed, a cellular ceramic, and a sintered metal.

The physical mechanism of local thermal equilibrium is explained as follows. The conduction heat transfer becomes dominant in a porous medium as either $Pr_{\mathrm{eff,f}} \to 0$, $Re_{d_p} \to 0$, or $Da \to 0$. In this case, the Nusselt number based on the interstitial heat transfer coefficient can be expressed in the form of

$$Nu = A + BPr^c Re_{d_p}^d \tag{35}$$

and approaches A which physically means that conduction is dominant. Then, the criterion expressed as Eq. (11) is met. So the assumption of local thermal equilibrium is always valid in all porous media when the conduction heat transfer mode is dominant, which coincides with what was previously suggested by other investigators [5–8].

On the other hand, the convection heat transfer is dominant as $Re_{d_p} \to \infty$. In this case, if the Nusselt number based on the interstitial heat transfer coefficient of a porous medium such as a sintered metal and a cellular ceramic is proportional to the Reynolds number with a power greater than 1, the Nusselt number is expressed as

$$Nu = A + BPr^c Re_{d_p}^d, \quad 0 < c < 1, \ d > 1 \tag{36}$$

By substituting Eq. (36) into Eq. (11) the criterion is expressed as

$$Pr_{\mathrm{eff,f}} Re_{d_p} Da^{1/2} \frac{\varepsilon}{A + BPr^c Re_{d_p}^d} \ll 1, \quad 0 < c < 1, \ d > 1 \tag{37}$$

The criterion expressed as Eq. (37) is met as $Re_{d_p} \to \infty$. Therefore, the assumption of local thermal equilibrium is valid even when the convection effects are dominant in a porous medium where the Nusselt number based on the interstitial heat transfer coefficient is proportional to the Reynolds number with a power greater than 1.

5. Conclusion

In this work a general criterion for local thermal equilibrium is presented in terms of parameters of engineering importance which include the Reynolds number, the Darcy number, and the Prandtl number. For this, an order of magnitude analysis is performed for the case when the effect of convection heat transfer is dominant in a porous structure. The criterion proposed in this study is more general than the previous criterion suggested by Carbonell and Whitaker [5], because the latter is applicable only when conduction is the dominant heat transfer mode in a porous medium while the former can be applied even when convection heat transfer prevails.

In order to validate the proposed criterion, a microchannel heat sink with an impinging jet is used. The effects of the Darcy number, the Prandtl number, and the Reynolds number on local thermal non-equilibrium in the microchannel heat sink with an impinging jet is well-predicted by the criterion. In addition, the proposed criterion is also confirmed with the existing experimental and numerical results for convection heat transfer in various porous materials that include some of the parameters used in the criterion such as a microchannel heat sink with a parallel flow, a packed bed, a cellular ceramic, and a sintered metal. The results presented in this study show that the assumption of local thermal equilibrium is always valid in all porous media when the conduction heat transfer mode is dominant. In other types of porous media such as sintered metals and cellular ceramics for which the interstitial heat transfer coefficient is proportional to the Reynolds number with a power greater than 1, the assumption of local thermal equilibrium is also valid when the convection heat transfer mode is dominant. Therefore, the proposed criterion, Eq. (11), can be used to check the validity of local thermal equilibrium in a porous medium when both convection and conduction are present.

Acknowledgements

This work was supported by Center for Electronic Packaging Materials of Korea Science and Engineering Foundation.

References

[1] A. Bejan, Editorial, J. Porous Media 1 (2000) i–ii.
[2] J.C.Y. Koh, R. Colony, Heat transfer of microstructure for integrated circuits, Int. Comm. Heat Mass Transfer 13 (1986) 89–98.
[3] S.J. Kim, D. Kim, Forced Convection in microstructure for electronic equipment cooling, ASME J. Heat Transfer 121 (1999) 635–645.
[4] S.J. Kim, D. Kim, D.Y. Lee, On the local thermal equilibrium in microchannel heat sink, Int. J. Heat Mass Transfer 43 (2000) 1735–1748.
[5] R.G. Carbonell, S. Whitaker, Heat and mass transfer in porous media, in: J. Bear, M.Y. Corapcigolu (Eds.), Fundamentals of Transport Phenomena in Porous Media, Martinus Nijhoff, 1984, pp. 121–198.
[6] S. Whitaker, Improved constraints for the principle of local thermal equilibrium, Ind. Eng. Chem. Res. 30 (1991) 983–997.
[7] M. Quintard, S. Whitaker, Local thermal equilibrium for transient heat conduction: theory and comparison with numerical experiments, Int. J. Heat Mass Transfer 38 (1995) 2779–2796.
[8] M. Quintard, S. Whitaker, Theoretical modeling of transport in porous media, in: K. Vafai (Ed.), Handbook of Heat Transfer in Porous Media, first ed., M. Decker, New York, 2000 (Chapter 1).
[9] K. Vafai, A. Amiri, Non-darcian effects in confined forced convective flows, in: B. Ingham (Ed.), Transport Phenomena in Porous Media, first ed., Elsevier Science, Amsterdam, 1998, pp. 313–329.
[10] A. Amiri, K. Vafai, Analysis of dispersion effects and non-thermal equilibrium, non-Darcian, variable porosity incompressible flow through porous media, Int. J. Heat Mass Transfer 30 (1994) 939–954.
[11] D.A. Nield, Effects of local thermal nonequilibrium in steady convective processes in a saturated porous medium: forced convection in a channel, J. Porous Media 1 (1998) 181–186.
[12] D.A. Nield, A.V. Kuznetsov, Local thermal nonequilibrium effects in forced convection in a porous medium channel: a conjugate problem, Int. J. Heat Mass Transfer 42 (1999) 3245–3252.
[13] D.Y. Lee, K. Vafai, Analytical characterization and conceptual assessment of solid and fluid temperature differentials in porous media, Int. J. Heat Mass Transfer 42 (1999) 423–435.
[14] M. Kaviany, Principles of Heat Transfer in Porous Media, second ed., Springer, Berlin, 1995 (Chapter 3).
[15] F.M. White, Viscous Fluid Flow, second ed., McGraw-Hill, New York, 1991 (Chapter 3).
[16] N. Wakao, S. Kaguei, T. Funazkri, Effect of fluid dispersion coefficients on particle-to-fluid heat transfer coefficients in packed beds, Chem. Engng. Sci. 34 (1979) 325–336.
[17] K.K. Kar, A. Dybbs, Internal heat transfer coefficient of porous metals, in: ASME Proceedings of the Winter Annual Meeting, Phoenix, Az, 1982, pp. 81–91.
[18] V. Maiorov, L. Vasiliev, V.M. Polyaev, Porous heat exchangers classification application, J. Engng. Phys. 47 (1984) 1110–1123.
[19] X. Fu, R. Viskanta, J.P. Gore, Measurement and correlation of volumetric heat transfer coefficients of cellular ceramics, Exp. Therm. Fluid Sci. 17 (1998) 285–293.